

Problem 1.11

Find the gradients of the following functions:

(a) $f(x, y, z) = x^2 + y^3 + z^4$.

(b) $f(x, y, z) = x^2 y^3 z^4$.

(c) $f(x, y, z) = e^x \sin(y) \ln(z)$.

Solution

Apply the gradient operator to each of the given functions.

$$\begin{aligned}\nabla f(x, y, z) &= \left(\hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) f(x, y, z) \\ &= \hat{\mathbf{x}} \frac{\partial f}{\partial x} + \hat{\mathbf{y}} \frac{\partial f}{\partial y} + \hat{\mathbf{z}} \frac{\partial f}{\partial z}\end{aligned}$$

Part (a)

Here $f(x, y, z) = x^2 + y^3 + z^4$.

$$\begin{aligned}\nabla f(x, y, z) &= \hat{\mathbf{x}} \frac{\partial}{\partial x} (x^2 + y^3 + z^4) + \hat{\mathbf{y}} \frac{\partial}{\partial y} (x^2 + y^3 + z^4) + \hat{\mathbf{z}} \frac{\partial}{\partial z} (x^2 + y^3 + z^4) \\ &= \hat{\mathbf{x}}(2x) + \hat{\mathbf{y}}(3y^2) + \hat{\mathbf{z}}(4z^3)\end{aligned}$$

Part (b)

Here $f(x, y, z) = x^2 y^3 z^4$.

$$\begin{aligned}\nabla f(x, y, z) &= \hat{\mathbf{x}} \frac{\partial}{\partial x} (x^2 y^3 z^4) + \hat{\mathbf{y}} \frac{\partial}{\partial y} (x^2 y^3 z^4) + \hat{\mathbf{z}} \frac{\partial}{\partial z} (x^2 y^3 z^4) \\ &= \hat{\mathbf{x}}(2x y^3 z^4) + \hat{\mathbf{y}}(3x^2 y^2 z^4) + \hat{\mathbf{z}}(4x^2 y^3 z^3)\end{aligned}$$

Part (c)

Here $f(x, y, z) = e^x \sin(y) \ln(z)$.

$$\begin{aligned}\nabla f(x, y, z) &= \hat{\mathbf{x}} \frac{\partial}{\partial x} [e^x \sin(y) \ln(z)] + \hat{\mathbf{y}} \frac{\partial}{\partial y} [e^x \sin(y) \ln(z)] + \hat{\mathbf{z}} \frac{\partial}{\partial z} [e^x \sin(y) \ln(z)] \\ &= \hat{\mathbf{x}}[e^x \sin(y) \ln(z)] + \hat{\mathbf{y}}[e^x \cos(y) \ln(z)] + \hat{\mathbf{z}} \left[\frac{e^x \sin(y)}{z} \right]\end{aligned}$$